Trajectories of passenger cars, vans and trucks have been classified into their lanes (different colors) at an intersection in Lower Saxony, Germany. This data were used to learn a model of human-driven vehicles.

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Mathematical Optimization and Machine Learning for Efficient Urban Traffic

Traffic jams cause economical damage which has been estimated between 10 and 100 billion Euros per year in Germany [6], also due to inefficient urban traffic [5]. It is currently open how the situation will change with upcoming technological advances in autonomous and electric mobility. On the one hand, autonomous cars may lead to an increased number of vehicles on the road with implied consequences. On the other hand, the availability of Vehicle2X (V2X) communication and smart algorithms might make the traffic flow more efficient, especially at natural bottlenecks such as urban traffic-light-controlled intersections.

To be able to quantify this anticipated potential to reduce waiting time, energy consumption, and CO_2 emissions, we developed mathematical models and tailored optimization algorithms. Mathematically optimal solutions provide bounds on what could be achieved. This versatile tool can be used to analyze a large variety of scenarios, including infrastructure investments, changes of traffic-light legislation, or the interplay between humans and autonomous vehicles. Numerical results indicate that the performance indicators time, energy, and emissions could be concurrently reduced by almost 50%. Potentially, the same models and algorithms might be the basis for future traffic control systems.

To calculate optimal switching of traffic lights and optimal autonomous driving of participants, we have been developing a mixed-integer optimization model and a variety of techniques that allow an efficient computation. Scenarios include fully-autonomous as well as mixed traffic, which leads to the additional challenge of incorporating realistic and uncertain human driving behavior into the model. To this end, we have been combining methods from different areas such as discrete and continuous mathematical optimization, control theory, and machine learning. Parts of the derived algorithms were successfully implemented and tested in a car of our industrial project partner Volkswagen Aktiengesellschaft.

Partners

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Industrial challenge and motivation

Urban traffic intersections are bottlenecks for an efficient traffic-flow and it is an economically and environmentally important question, how to improve the performance. There are two obvious possibilities to tackle the challenge: First, the optimization of the traffic light systems, and second to improve the behavior of the autonomous vehicles and human operated vehicles. Both degrees of freedom can be combined into a single, centralized optimization problem. This allows to estimate the potential for improvements of selected measures, such as introduction or modification of traffic light regulations, or the usage of optimized algorithms for autonomous driving. In addition, the approach allows to determine the capacity of a road network under maximum possible coordination, which is not only of interest for infrastructure planning purposes, but also serves as a benchmark for decentralized and other heuristic approaches. In order to obtain valid bounds, however, one needs to solve the resulting models to global optimality. This is very challenging even on a very simple network and short time horizons. With standard approaches for moving horizon nonlinear model predictive control (NMPC) runtimes significantly exceed real-time requirements.
From a modeling point of view, several challenges arise. The idealized central optimization model has to be extended in several ways to cope with different types of traffic that we expect in the near future. We address the following issues for a comprehensive investigation of efficient future traffic at urban intersections:

- A centralized optimization approach that gives an idea of how a fully automatized and centrally coordinated traffic would look like, and what could be theoretically achieved. Given the complexity of the optimization problem, mathematical modeling and optimization algorithms need to be developed.
- Mixed-models that include human drivers, bicycles, and pedestrians. Such submodels must be based on insight extracted from data. Subsequently, these subproblems need to be included in the centralized optimization problem formulation with new algorithms (e.g., NMPC).
- An investigation should clarify which effects can already be obtained with technology already available and without huge investments. For example, driving assistant systems might facilitate some of the ideas from central optimization simply based on the knowledge of the traffic light switching. Such a system could be realized with V2X communication and NMPC, and may or may not incorporate heuristic central optimization of traffic lights.

Mathematical research

Centralized optimization The goal of the central optimization approach is to determine the influence of optimizing certain degrees of freedom on the resulting traffic in terms of efficiency of the traffic flow (waiting times, travel times), but also on environmental quantities like fuel consumption and CO_2 emissions. The first step is to develop a suitable model to describe the movement of cars through the network and their interaction with other cars and the traffic lights. The basic structure of our scenario is as follows: Straight roads intersect in a single intersection. Each road consists of two lanes running in opposite directions. On each lane, a traffic light is installed in front of the intersection that regulates the traffic on its lane. We use a simple longitudinal model to describe the vehicle dynamics $x = (s, v, a)$ consisting of position, velocity and acceleration. The biggest challenge in modeling traffic at intersections is preventing collisions between cars. There are two types of possible collision situations we have to address in our modeling: First, we have to make sure that vehicles driving on the same lane do not collide, and second, we have to prevent collisions inside the intersection area, where vehicles from different lanes have to coordinate. Collision prevention constraints outside the intersection area are of the form

$$s_{c',t} - s_{c,t} \geq l_{c'} + g_c \quad \forall c \in C,\ c' \in C_c^{\text{pred}},\ t \in T, \tag{1}$$

ensuring a sufficient distance between two vehicles c and c'. Inside the intersection area traffic lights coordinate the traffic in such a way that vehicles can safely cross the intersection. Traffic lights are modeled via binary variables, which can take the binary-encoded states red and green. The idea is that if the intersection area is occupied by a vehicle, the traffic light should give red to other lanes that can potentially cause a collision, essentially blocking the intersection for their vehicles. Indicator formulations track if a vehicle is inside the intersection area and enable or disable such constraints based on the location of the car. Special care is needed to adapt (1) for modeling turning vehicles since predecessor-successor relations may be invalidated after a turning maneuver—details of which are subject to optimization.

Our main goal is to optimize traffic flow, which could be interpreted as reducing the overall travel time of all vehicles for reaching their destination. As the time horizon is fixed, minimizing traveling time is similar to maximizing the covered distance at the end of the time horizon in our scenario. Therefore, we maximize the sum of the driven distances of all cars at the last time step N:

$$\begin{aligned} \max \quad & \sum_{c \in C} s_{c,N} \\ \text{s.t.} \quad & x \in X \end{aligned}$$

This optimization problem is formulated as a mixed integer linear program (MILP) and yields a solution which is optimal in terms of traffic flow, but not necessarily in terms of fuel efficiency or CO_2 reduction. To achieve this objective, we follow a two-step approach. First, we optimize our model in terms of traffic flow, then we solve the same model with a changed objective function while demanding the same obtained traffic flow from the first optimization via an additional constraint and fixing all binary variables. That way, only a quadratic program (QP) has to be solved in the second step.

Table 1 shows averaged results over 5 example instances with relatively high traffic density. Centrally optimized traffic leads to almost no waiting time in case of completely free traffic lights. However, also the more realistic case of controllable traffic lights that are bound to certain regulations on phase lengths leads to a significant reduction, which is also accompanied by reductions in fuel consumption and CO_2 emission when compared to non-optimized traffic. Further computational results can be found in [3].

density [Cars/min]	T [s]	dt [s]	l [s]	waiting time [s]	travel time [s]	fuel [l/100km]	CO_2 [g/km]
20.95	60	0.5	10				
real-world simulation				14.6659	57.8923	10.6744	248.293
fixed traffic lights				8.14332	39.1992	9.18923	213.801
regulated traffic lights				7.39298	37.8572	8.95977	208.473
free traffic lights				0.139659	26.8419	6.41469	149.234

Table 1: Computational results for traffic with different degrees of optimization. All measurements are per vehicle and are geometric means over five randomized test instances. From [3, Table 4].

Decentralized optimization of mixed traffic Another part of investigation is concerned with the efficiency of mixed traffic, namely, a situation when human drivers act alongside autonomous vehicles. This drives a need for modeling human drivers' behavior. Such models can be based on first principle modelling, machine learning or a combination of both, e.g. combining a bicycle model $f(\cdot)$ [2] with neural networks $h(\cdot)$:

$$
\begin{aligned}
z(k+1) &= f(z(k)) + h(\sigma(\cdot), z(k)) \\
\sigma(\xi) &= \frac{1}{1+e^{-\theta \cdot \xi}}.
\end{aligned} \tag{2}
$$

The parameters θ of the neural network are learned during the training phase using the features ξ. The future state $z(k+1)$ (vehicle position and velocity) of the human driven vehicle is then predicted for the next time steps using the neural network. By solving the optimal control problem for an autonomous car (3) (decentral optimization and non-connected vehicles), it crosses the intersection as soon and fast as possible, while keeping a minimum distance from a human driven vehicle at all times.

$$
\begin{aligned}
&\min_{\hat{u}(\cdot),\, \hat{s}(\cdot)} \; J(\hat{u}(\cdot),\, \hat{x}(\cdot)) \\
&s.t. \quad J(\cdot) = \sum_{l=k}^{k+N-1} \left(\| \hat{u}(l) \|_{R_1}^2 + \| \Delta \hat{u}(l) \|_{R_2}^2 \right) \\
&\qquad\qquad + \sum_{l=k+1}^{k+N} \left(\| \hat{x}(l) - x_{\text{ref}}(s) \|_{Q_1}^2 + \| \hat{s}(l) \|_{Q_2}^2 \right) \\
&\hat{x}(k+i) = f(\hat{x}(\cdot),\, u(\cdot)), \quad \hat{x}(k) = x(k) && \text{(3a)} \\
&\hat{\mathbf{z}}_{j,m}(k+i) = h(\theta_m, \mathbf{z}_j(\cdot),\, \hat{\mathbf{z}}_{j,m}(\cdot)), && \text{(3b)} \\
&d(\hat{x}(k+i),\, z_{j,m}(k+i)) \geq \epsilon_d, \quad \forall i \in [0,\, N], && \text{(3c)} \\
&\Delta u = \hat{u}(k) - \hat{u}(k-1), \quad \Delta u(k) = 0 && \text{(3d)}
\end{aligned}
$$

The geometric path $x_{\text{ref}}(s)$ of the autonomous car is a priori defined based on the layout of the intersection, while the position along the path $\hat{s}$ and the acceleration $\hat{u}$ of the autonomous car are determined online by solving the optimal control problem (3). The path position $\hat{s}$ and the optimal acceleration profile $[\hat{u}(k),\, \hat{u}(k+1),\, ,\dots,\, \hat{u}(k+N)]$ are constrained by the predicted position of the human driven vehicle (3b) and the minimum distance ϵ_d (3c) between the autonomous car $\hat{x}(k+i)$ and the human driven vehicle $\hat{z}(k+i)$ predicted through the neural network (2). The state $\hat{x}(k+i)$ of the autonomous car at time step $k+i$ is predicted and optimized based on the optimal acceleration profile $\hat{u}(\cdot)$ and the car model (3a). By setting $R_2 > 0$ one can incorporate the notion of travel comfort for the occupants of the autonomous car. It aims to reduce the changes in acceleration profile (3d) of the autonomous car, which should also reduce fuel consumption. However, this is not necessary with respect to safety.

The decentralized approach was implemented for a single autonomous vehicle, which does not depend on wireless communication, as e.g. V2X or V2V. Preliminary results show that based on the uncertainty of the human driven vehicle model and the tuning parameters (R_1, R_2, Q_1, Q_2, ϵ_d) the autonomous vehicle crosses the intersection in less time and with smaller distance between the vehicles than human drivers.

We validated the decentralized optimal controller using first principle models and a neural network to model the human driving behavior (2) in [1]. A sketch of the controllers' behavior is shown in Figure 2. When approaching an intersection the autonomous car gains data on the human driven car. The autonomous car predicts the trajectory of the human driven vehicle (red arrows) and its probability (thickness of the red arrows). Based on these predictions, the acceleration (thickness of yellow arrows) of the autonomous car is optimized and results in the planned trajectory (yellow arrow) of the autonomous vehicle. Far away from the intersection the possibilities (turning left or right or going straight) of the human driven car are equally likely and result in multiple predicted trajectories—one for each possibility. Due to updated information the

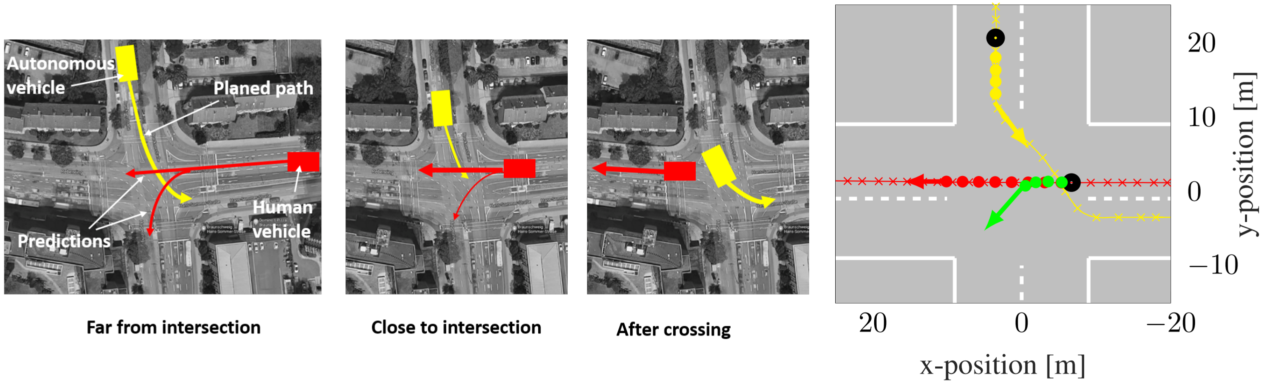

Figure 2: Intersecting trajectories of the autonomous vehicle (yellow) and the human driven vehicle (red and green). A sketch of the decentral optimization concept over time can be seen on the left side, while the right side shows a corresponding simulation example based on real data.

autonomous car removes less probable possibilities and adapts its own acceleration until it safely turns to cross the intersection behind the human driven car. In the simulation example on the right side of Figure 2 the path of the autonomous vehicle is depicted by a yellow line. The simulation shows the planned (yellow dots) and predicted paths (red and green) at time step $k = 50$, i.e. $t = 5\,\mathrm{s}$. The future position of the human driven vehicle is predicted for two possible modes. These are going straight from east to west (red dots with direction arrow) or turning left from east to south (green crosses with direction arrow). Both predicted trajectories of the human driven vehicle are considered as constraints in the optimal control problem (3), i.e. the minimum distance $\epsilon_d = 5\,\mathrm{m}$ should be kept at all time steps $k + i$ during the prediction horizon $i = [0, N]$. This leads to an optimal control input $\hat{u}(\cdot)$ for the autonomous car and the predicted position trajectory (yellow dots) at $k = 50$, shown on the right side of Figure 2. It can be seen that the minimum distance is kept for all predicted time steps at $k = 50$. Furthermore, the final trajectories for the autonomous car (yellow crosses) and the human driven car (red crosses) are shown as thin lines.

Decentralized optimization with V2X communication Finally, a third approach realizes a combination of decentralized optimization of the velocity of each car together with a centralized control of traffic light states. Cars register with the system when approaching a traffic light, which then greedily assigns transition trajectories on a first-come-first-serve basis with respect to some global objective. This is done after solving a simple mixed-integer program where the trajectories of all previously registered cars are fixed. If no feasible solution can be found, one or multiple correction steps are triggered which allow reassessment of the trajectories of other cars. We evaluated this heuristic method employing central optimization as a benchmark. Despite its computational simplicity, the decentralized optimization approach allowed us to reach near-optimal traffic efficiency in many instances, in particular for low to medium traffic densities.
As a preliminary step for practical testing, we considered the problem of optimally approaching a traffic light knowing its switching times, which can be realized by a V2X-based velocity controller.

Implementation

The Research Group of Volkswagen Aktiengesellschaft brought up the questions and shared their experience gained in former related projects. They also provided real-world data from cars and traffic infrastructure, and built a prototypical car, which allowed to implement and test one of the developed algorithms in real-world traffic. Parts of the real-world data were gathered from cameras on a research junction in Brunswick (see title picture). This junction is a 4-way traffic-light-controlled intersection. The data set includes the traffic-light signals and all traffic participants, which were categorized to cars, trucks, vans, motorcycles, bicycles, pedestrians, and railway vehicles. The data set consists of over 140.000 trajectories of motorized vehicles and includes position trajectories with time-stamps and vehicle dimension separately for each car, while the velocity and acceleration trajectory were interpolated using the position and time-stamps. The researchers at the Otto von Guericke University Magdeburg took the lead in the design and implementation of models and solving procedures. Especially their background in control theory—also in an automotive setting—was very fruitful for the project. The shared work of the partners was embedded in several PhD-theses and in the Research Training Group GK 2297 on mathematical complexity reduction.

We considered several related approaches, which were all based on mathematical optimization. First, a velocity controller was implemented, which aims for the green phase of a traffic-light. Key technology for this is the wireless communication via V2X between the traffic-light and an approaching car. Multiple times per second, data about the current state of the traffic-lights as well as upcoming phase switches are exchanged. This allows the calculation of velocity trajectories in the car with the aim of passing the stopping line as soon as the light switches to green and as close as possible to the driver's preferred velocity. If possible, full stops are avoided. Figure 3 gives an example illustration of such a system that has been implemented based on work from our project by Volkswagen Aktiengesellschaft. Following this, the other approaches described above were developed.

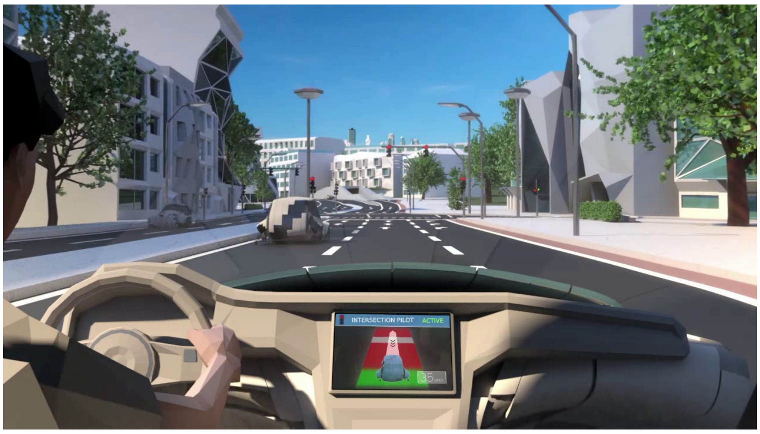

Figure 3: Driver's view when approaching a traffic-light. The driver-assistance system ensures an automatized passage after the light switches to green without having to stop.

Industrial relevance and Summary

The majority of research and engineering efforts concerning connected autonomous vehicles (CAV) focuses on safety matters. Talking about sustainable mobility and mobility services which aim to use as few (public) resources as possible (e.g. space, time, air, and noise pollution), an efficient and smooth traffic flow needs to be achieved. Mathematical studies have shown that even single CAVs can have a positive impact on overall traffic flow [7]. Thus, research on optimizing the individual and collective driving behavior of CAVs is of current interest. In fact, human drivers are already able to adapt to different traffic situations and to implicitly perform group maneuvers. A first step of development needs to make CAVs as efficient as current human driven vehicles. Further potential beyond this should be unlocked in future development of CAVs, for which this project carves a path.

One major result was the evaluation of the effects on traffic-flow for all three approaches. Best results were naturally achieved by the central optimization model. A reduction in the mean waiting time of all cars of up to 99% could be achieved. Also, the average fuel consumption could be reduced by 39%. Furthermore, the model allows to compare the influence of different parameters, such as traffic density, traffic light scheme, share of automated vehicles or speed limit, on the traffic efficiency. Surprisingly, the hybrid method achieved similar outcomes although necessary solving times are much shorter and thus offer a practical implementation. Finally, the V2X-based velocity controller led to 28% less waiting times and 19% less fuel consumption. Furthermore, it was implemented and successfully tested in a real car. Test runs in real-world traffic in the cities of Brunswick and Düsseldorf confirmed the practicability of the assistance system.

Patents

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