

Proseminar Konvexe Optimierung

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- Ziel: Vertiefung einiger Themen der Optimierung, insbesondere der Quadratischen Programmierung (QP)

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- Folgend sind Literaturangaben gegeben, die den Einstieg erleichtern. Es wird empfohlen, eigenständig weitere Quellen zu identifizieren und zu nutzen!

- 1 *Problem formulation and applications*
- 2 *Optimality conditions for QPs*
- 3 *QP duality theory*
- 4 *The ellipsoid method*
- 5 *Direct linear algebra for equality-constrained QPs*
- 6 *Iterative linear algebra for large-scale QPs*
- 7 *Gradient projection methods for bound-constrained QPs*
- 8 *Active-set methods for inequality-constrained QPs*
- 9 *Interior-point methods for inequality-constrained QPs*
- 10 *Homotopy methods for parametric QPs*
- 11 *Linear complementarity problems (LCPs) and their relation to QPs*
- 12 *Copositive programming*
- 13 *The S-Lemma*

1. Problem formulation and applications

Contents:

- Problem formulation
- Portfolio optimization
- Model predictive control (MPC)
- Sequential quadratic programming (SQP) for nonlinear optimization

Literature references:

- J. Nocedal, S.J. Wright. Numerical Optimization, Springer, 2006. Ch. 16.1, pp. 449–450, 529–534.
- C. Geiger, C. Kanzow. Theorie und Numerik restringierter Optimierungsaufgaben. Springer, 2002. Ch. 5.1, pp. 197–198.
- A. Alession, A. Bemporad. A Survey on Explicit Model Predictive Control. In: Nonlinear Model Predictive Control. Lecture Notes in Control and Information Sciences, Springer, 2009, Volume 384, pp. 345–347, 362–363.

2. Optimality conditions for QPs

Contents:

- Mathematical derivation of necessary and sufficient conditions of optimality for convex and nonconvex QPs
- Degeneracy
- Basis for following numerical methods

Literature references:

- P.E. Gill, W. Murray, M.H. Wright. **Practical Optimization**. Elsevier, 2004. Ch. 3, pp. 59–77.
- C. Geiger, C. Kanzow. Theorie und Numerik restringierter Optimierungsaufgaben. Springer, 2002. Ch. 2.2, pp. 40–55, Ch. 5, pp. 197–205
- J. Nocedal, S.J. Wright. Numerical Optimization, Springer, 2006. Ch. 16.4, pp. 465–467.

3. QP duality

Contents:

- Primal and dual QP, convex and nonconvex case
- Duality theorems (Franke-Wolfe, Dorn)

Literature references:

- R.W. Cottle, J.-S. Pang, R.E. Stone. The Linear Complementarity Problem, Classics in Applied Mathematics 60, SIAM, 2009. pp. 113–117
- O.L. Mangasarian. Duality in quadratic programming. In ‘Nonlinear Programming’, chapter 8-2, pp. 123–126. McGraw-Hill, New York, USA, 1969. Reprinted as Classics in Applied Mathematics 10, SIAM, Philadelphia, USA, 1994.
- J.J. Júdice. The duality theory of general quadratic programs. Portugaliae Mathematica, 42, 113–121, 1984.

4. The ellipsoid method

Contents:

- The ellipsoid method
- Implications on complexity of linear/quadratic programming

Literature references:

- Vorlesungsfolien Wintersemester, Kapitel 7
- R. G. Bland, D. Goldfarb, and M. J. Todd. The ellipsoid method: A survey. *Operations Research*, 29(6):1039–1091, 1981.
- S. Rebennack, *Ellipsoid method*, 2007.

5. Direct linear algebra for equality constrained QPs

Contents:

- Range and null space methods
- Symmetric indefinite decomposition

Literature references:

- J. Nocedal, S.J. Wright. Numerical Optimization, Springer, 2006. Ch. 16.1, 16.2, pp. 451–459.
- M. Benzi, G.H. Golub, J. Liesen. Numerical solution of saddle point problems. Acta Numerica 14, pp. 1–137, 2005. Focus pp. 30–34, 40–43.

6. Iterative linear algebra for equality constrained QPs

Contents:

- Spectral properties of saddle-point matrices
- Iterative solution methods (projected CG, MINRES)
- Preconditioning

Literature references:

- J. Nocedal, S.J. Wright. Numerical Optimization, Springer, 2006. Ch. 16.3, pp. 459–463.
- M. Benzi, G.H. Golub, J. Liesen. Numerical solution of saddle point problems. Acta Numerica 14, pp. 1–137, 2005. Focus pp. 14–29, 43–96.

7. Bound constrained QPs

Contents:

- Gradient projection method for QPs with only bound constraints

Literature references:

- J.J. Moré and G. Toraldo. On the solution of large quadratic programming problems with bound constraints. *SIAM Journal on Optimization*, 1(1), 93–113, 1991.
- J.J. Moré and G. Toraldo. Algorithms for bound constrained quadratic programming problems. *Numerische Mathematik*, 55(4), 377–400, 1989.
- J. Nocedal, S.J. Wright. *Numerical Optimization*, Springer, 2006. Ch. 16.7, pp. 485–490.

8. Active set methods for inequality constrained QPs

Contents:

- Primal active set method for QPs with linear inequalities
- Dual active set method for convex QPs with linear inequalities
- Updates for matrix decompositions

Literature references:

- J. Nocedal, S.J. Wright. Numerical Optimization, Springer, 2006. Ch. 16.5, pp. 467–480.

9. Interior point methods for inequality constrained QPs

Contents:

- Interior point method for QPs with linear inequalities
- Numerical treatment of saddle-point systems

Literature references:

- Vorlesungsfolien Wintersemester, Kapitel 7
- J. Nocedal, S.J. Wright. Numerical Optimization, Springer, 2006. Ch. 16.6, pp. 480–485.
- M. Benzi, G.H. Golub, J. Liesen. Numerical solution of saddle point problems. Acta Numerica 14, pp. 1–137, 2005. Focus pp. 12–14.

10. Homotopy methods for parametric QPs

Contents:

- Parametric QPs
- Primal-dual active set method for convex parametric QPs

Literature references:

- M.J. Best. An Algorithm for the Solution of the Parametric Quadratic Programming Problem. Applied Mathematics and Parallel Computing – Festschrift for Klaus Ritter, Physica-Verlag, 1996, ch. 3, pp. 57–76.

11. Linear complementarity problems for QPs

Contents:

- Linear complementarity problems (LCP)
- Optimality conditions
- Connection with Quadratic Programming
- Dantzig-Wolfe algorithm for LCP and QP

Literature references:

- P. Wolfe. The simplex method for quadratic programming. *Econometrica* 27, pp. 382–398, 1959.
- R.W. Cottle, J.-S. Pang, R.E. Stone. The Linear Complementarity Problem, *Classics in Applied Mathematics* 60, SIAM, 2009. pp. 3–5, 23, 29, 113–117, 138.

12. Copositive Programming

Contents:

- Definition
- Applications
- Geometric properties
- Testing copositivity

Literature references:

- M. Duer. Copositive Programming – a Survey. In: M. Diehl, F. Glineur, E. Jarlebring, W. Michiels (Eds.), Recent Advances in Optimization and its Applications in Engineering, Springer 2010, pp. 3-20.
- S. Boyd, L. Vandenberghe. Convex Optimization. Cambridge University Press, 2004.

13. The S-Lemma

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- Linear complementarity problems (LCP)
- Optimality conditions
- Connection with Quadratic Programming
- Dantzig-Wolfe algorithm for LCP and QP

Literature references:

- I. Polik, T. Terlaky. A Survey of the S-Lemma. *SIAM Review*, Vol. 49 (3), pp. 371–418, 2007.